Line Width and Bandwidth of Millimeter-Wave Resonance Isolators*

P. VILMUR†, STUDENT MEMBER, IRE, AND K. ISHII†, MEMBER, IRE

Summary—**A theoretical derivation is made of bandwidth as a function of resonant frequency of a single crystal ferrite resonance isolator at millimeter wavelengths. The derivation takes into account the ferrite isolator as a bounded system. Using the derived relation of bandwidth and resonant frequency, and Kittel's relation between resonant frequency and applied field, equations are derived which relate line width to resonant frequency, line width to the applied magnetic field, and line width to frequency bandwidth. These resulting equations are compared with experimental data obtained with a single crystal barium ferrite isolator from 58 to 59 kMc. The theoretical relations agreed closely with the experimental data within the accuracy of the measuring equipment at these frequencies. In general, the results showed that for small frequency ranges (1 kMc) bandwidth and line width increase almost linearly with frequency, bandwidth and line width are linearly related, and line width is a fairly complicated but increasing function of applied field.**

Introduction

RELATIONS between the various parameters in common use to describe the operation of ferrite resonance isolators, such as line width as a function of frequency, have been developed by a number of authors.[1-3] In many cases, these parameters are based on the assumption that the ferrite is unbounded or is spherical in shape. These simplifications have led to fairly compact equations relating, for example, line width to resonant frequency. But, these equations concern the behavior of the ferrite material itself and do not take into account the totally bounded system, consisting of the piece of ferrite coupled with a specific microwave circuit. So, it has been found in practice that experimental results do not always agree with these equations. In particular, some of the experimental data obtained for a single crystal ferrite resonance isolator at 58 to 59 kMc could not be explained by any theoretical relations known to us.

It is the purpose of this work to develop relations between bandwidth and resonant frequency, line width and resonant frequency, line width and bandwidth, and finally line width and applied field. These results will be compared to experimental data obtained in the millimeter wave region.

* Received by the PGMTT, September 11, 1961; revised manuscript received, October 25, 1961. This research is supported by the A. O. Smith Corp. Research Grant to Marquette University, Milwaukee, Wis.

† Department of Electrical Engineering, Marquette University, Milwaukee, Wis.

[1] R. F. Soohoo, "Theory and Applications of Ferrites," Prentice-Hall, Inc., Englewood Cliffs, N. J., pp. 72–74, 132–136; 1960.

[2] B. Lax, "Frequency and loss characteristics of microwave ferrite devices" PROC. IRE, vol. 44, pp. 1368–1385; October, 1956.

[3] J. K. Galt, "Losses in ferrites: Single-crystal studies," PROC. *IEE*, vol. 104, pt. B, suppl. No. 5, pp. 189–197; November, 1957.

The approach that was made to the problem of finding how bandwidth is related to resonant frequency was different. Using perturbation theory, Lax[2] derived an equation for ferrite isolators, relating attenuation and the mounting position of the ferrite in the waveguide. In this paper, Lax's equation was extended by solving it with respect to frequency to obtain a specific relation between bandwidth and resonant frequency. Since Lax's equation considers the ferrite and the waveguide as a coupled system, this newly derived relation also considers the ferrite isolator as a totally bounded system, taking into account the mounting position of the ferrite in the waveguide and the shape of the ferrite. A similar approach to this problem, leading directly to a relation between bandwidth and resonant frequency, has not been found in the literature.

The derived relation between bandwidth was combined with Kittel's equation of resonant frequency and applied field[4] to obtain a relation between line width and resonant frequency. From the two newly obtained relations, a new relation between line width and bandwidth was derived.

Finally, the relation between line width and the applied field was derived using the results of line width and resonant frequency, and Kittel's equation.

These derived relations provided a firm basis to explain more realistically the behavior of millimeter wave resonance isolators. In particular, the relations between line width, resonant frequency, bandwidth, and applied field were especially useful since this line width describes the operation of a practical isolator and is not just a theoretical descriptive parameter for an unbounded ferrite crystal.

Bandwidth and Resonant Frequency

The purpose of this section is to derive an equation of bandwidth of millimeter wave resonance isolators using a single crystal ferrite.[5-8]

[4] C. Kittel, "On the theory of resonance absorption," *Phys. Rev.*, vol. 73, pp. 155–161; January, 1948.

[5] K. Ishii, J. Tsui, and F. Wang, "Millimeter wave field-displacement-type isolators with short ferrite strips," PROC. IRE, vol. 49, pp. 975–976; May, 1961.

[6] K. Ishii, J. Tsui, and F. Wang, "Effects of ferrite strip mounting positions on millimeter wave isolator characteristics," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-9, p. 362; July, 1961.

[7] F. Wang, K. Ishii, and J. Tsui, "Ferrimagnetic resonance of single crystal barium ferrite in millimeter wave region," *J. Appl. Phys.*, vol. 32, pp. 1621–1622; August, 1961.

[8] J. Tsui, "Field Displacement Millimeter-Wave Isolator," M.S. thesis, on file at Marquette University, Milwaukee, Wis.; 1961.

It has been shown by Lax[2] that the loss in a ferrite in the backward direction is

$$\alpha = C(S_1\beta^2\chi_{xx}'' + S_2\chi_{yy}'' + S_3\beta\chi_{xy}''), \tag{1}$$

where

C is a constant depending on the cross section of the ferrite loaded waveguide.

χ_{xx}'' and χ_{yy}'' are the imaginary parts of the diagonal components of the susceptibility tensor.

χ_{xy}'' is the imaginary part of the off diagonal component of the susceptibility tensor

$$\beta^2 = \omega^2\epsilon_0\mu_0 - k^2 \tag{2}$$

$$k = \frac{\pi}{a} \tag{3}$$

$$S_1 = \sin^2 kx_0 \tag{4}$$

$$S_2 = k^2 \cos^2 kx_0 \tag{5}$$

$$S_3 = k \sin 2kx_0 \tag{6}$$

x_0 is the ferrite position from the guide wall.

Putting in the value of the respective tensor susceptibilities, the equation becomes

$$\frac{\alpha}{C} = S_1(\omega^2\epsilon_0\mu_0 - k^2)\omega_m T \frac{\omega_r^2 - \omega^2 - 1/T^2 - 2\omega_r\cdot[\omega_0 + (N_y - N_z)\omega_m + \omega_a]}{T^2(\omega_r^2 - \omega^2 - 1/T^2)^2 + 4\omega_r^2}$$
$$+ S_2\omega_m T \frac{\omega_r^2 - \omega^2 - 1/T^2 - 2\omega_r[\omega_0 + (N_x - N_z)\omega_m + \omega_a]}{T^2(\omega_r^2 - \omega^2 - 1/T^2)^2 + 4\omega_r^2}$$
$$+ S_3\sqrt{\omega^2\epsilon_0\mu_0 - k^2}\,\frac{2\omega\omega_r\omega_m T}{T^2(\omega_r^2 - \omega^2 - 1/T^2)^2 + 4\omega_r^2}, \tag{7}$$

where

$\omega_m = 4\pi\gamma M$, M is the magnetization.

$\omega_0 = \gamma H_0$, γ is the gyromagnetic ratio, H_0 is the applied field.

T is a phenomenological relaxation time.

N_x, N_y, N_z, are the respective demagnetization factors.

$\omega_a = \gamma H_a$, H_a is the anisotropy field.

Multiplying through by the denominator of (7) and rearranging, the term on the left will be

$$\frac{\alpha}{C\omega_m T}\,[T^2\omega^4 - 2(\omega_r^2T^2 - 1)\omega^2 + \omega_r^2(\omega_r^2T^2 + 2) + 1/T^2]. \tag{8}$$

In this equation $T^2\omega_r^2 \gg 1$ for ferrite resonance isolator.[9] Using this simplification (8) becomes

$$\frac{\alpha}{C\omega_m T^3}\,[T^4\omega^4 - 2\omega_r^2\omega^2T^4 + \omega_r^4T^4]. \tag{9}$$

After multiplying through by the denominator of (7) and rearranging, the first term on the right becomes

$$-S_1[\epsilon_0\mu_0\omega^4 - (A^0\epsilon_0\mu_0 + k^2)\omega^2 + A^0k^2], \tag{10}$$

where

$$A^0 = \omega_r^2 + 1/T^2 + 2\omega_r[\omega_0 + (N_y - N_z)\omega_m + \omega_a].$$

The second term on the right will be

$$-S_2(\omega^2 - A^1) \tag{11}$$

where

$$A^1 = \omega_r^2 - 1/T^2 - 2\omega_r[\omega_0 + (N_x - N_z)\omega_m + \omega_a].$$

The third term on the right will be

$$-S_3\sqrt{\epsilon_0\mu_0}\,\omega_r\left(2\omega^2 - \frac{k^2}{\epsilon_0\mu_0}\right), \tag{12}$$

where the assumption was made that

$$\frac{k^2}{\omega^2\epsilon_0\mu_0} \ll 1.$$

Collecting together all terms in like powers of ω in (10), (11), and (12), the following equation results:

$$K_1\omega^4 + K_2\omega^2 + K_3 = 0 \tag{13}$$

where

$$K_1 = \frac{\alpha T}{C\omega_m} + S_1\epsilon_0\mu_0 \tag{14}$$

$$K_2 = -2\omega_r^2\frac{\alpha T}{C\omega_m} - S_1(A^0\epsilon_0\mu_0 + k^2) + S_2 - 2S_3\sqrt{\epsilon_0\mu_0}\,\omega_r \tag{15}$$

$$K_3 = \omega_r^4\frac{\alpha T}{C\omega_m} + S_1A^0k^2 - S_2A^1 + \frac{S_3k^2\omega_r}{\sqrt{\epsilon_0\mu_0}}. \tag{16}$$

[9] According to Lax,[2] approximate reverse loss to forward loss is given by $R \approx (2w_rT)^2$, where R is always much greater than one for ordinary isolators.

At $\omega=\omega_r$ the attenuation in the ferrite will be maximum. The 3 db attenuation points will be one-half this maximum since α is in units of nepers/unit length. Going back to (7) and putting in $\omega=\omega_r$ this equation becomes

$$\frac{\alpha_{\max}}{2C} = S_1\beta\frac{2\omega_m T[\omega_0+(N_y-N_z)\omega_m+\omega_a]}{4\omega_r} + S_2\frac{\omega_m T[\omega_0+(N_x-N_y)\omega_m+\omega_a]}{4\omega_r} + S_3\frac{\beta\omega_m T}{4} \tag{17}$$

where again it was assumed that $\omega_r^2T^2\gg1$. Putting (17) into (14) for α/C

$$K_1' = C_3\omega_r + C_4/\omega_r + C_5\sqrt{\omega_r^2+C_6} + C_1 \tag{18}$$

where

$$C_1 = S_1\epsilon_0\mu_0$$

$$C_3 = \frac{S_1T^2\epsilon_0\mu_0}{4}[\omega_0+(N_y-N_z)\omega_m+\omega_a]$$

$$C_4 = \frac{S_2T^2}{4}[\omega_0+(N_x-N_z)\omega_m+\omega_a] - \frac{S_1k^2T^2}{4}[\omega_0+(N_y-N_z)\omega_m+\omega_a]$$

$$C_5 = \frac{S_3\sqrt{\epsilon_0\mu_0}T^2}{4}$$

$$C_6 = \frac{-k^2}{\epsilon_0\mu_0}.$$

The "constants" C_1 through C_6 contain parameters such as T and ω_0, which are slightly frequency dependent. Because the frequency range considered in this paper is small, they are assumed to be constant. Similarly, putting (17) into (15),

$$K_2' = -2C_3\omega_r^3 - C_1\omega_r^2 - 2C_4\omega_r - 2C_5\omega_r^2\sqrt{\omega_r^2+C_6} - C_2 \tag{19}$$

where

$$C_2 = S_2 - S_1k^2\,.$$

C_1, C_3, C_4, C_5 are the same as in (18).

All but the first terms of $-S_1A^\circ\epsilon_0\mu_0$ and the term $-2S_3\sqrt{\epsilon_0\mu_0}\omega_r$ were dropped from (15) since they were negligible compared to the other terms at millimeter wavelengths.

Putting (17) into (16) K_3 will be

$$K_3' = C_3\omega_r^5 + C_4\omega_r^3 + C_2\omega_r^2 + C_5\omega_r^4\sqrt{\omega_r^2+C_6} \tag{20}$$

where

C_2, C_3, C_4, C_5, and C_6 are the same as in (19).

All but the first terms $S_1A^\circ k^2$ and $-S_2A^2$ were dropped from (16) since they tended to cancel each other at millimeter wavelengths. The term $S_3k^2\omega_r/\sqrt{\epsilon_0\mu_0}$ was dropped because it was negligible compared to the other terms in (16). For the value of attenuation at the 3 db points, (13) can be written

$$K_1'\omega^4 + K_2'\omega^2 + K_3' = 0. \tag{21}$$

The solutions of this equation are

$$\omega_1^2 = \frac{-K_2' - \sqrt{K_2'^2 - 4K_1'K_3'}}{2K_1'}, \tag{22}$$

$$\omega_2^2 = \frac{-K_2' + \sqrt{K_2'^2 - 4K_1'K_3'}}{2K_1'}, \tag{23}$$

where ω_1 and ω_2 are the radian frequencies at the 3 db points.

Subtracting (22) from (23),

$$\omega_2^2 - \omega_1^2 = \frac{\sqrt{K_2'^2 - 4K_1'K_3'}}{K_1'}. \tag{24}$$

For this equation bandwidths are very small (10 to 100 Mc) compared to the resonant frequency. Therefore, $\omega_2+\omega_1$ can be written as $2\omega_r$ and

$$\omega_2^2 - \omega_1^2 = 2\omega_r\Delta\omega, \tag{25}$$

where

$$\Delta\omega = \omega_2 - \omega_1 = \text{radian frequency bandwidth.}$$

Substituting (25) back into (24),

$$\Delta W = \frac{\sqrt{K_2'^2 - 4K_1'K_3'}}{2\omega_r K_1'}. \tag{26}$$

Substituting (18), (19), and (20) into (26) this equation reduces to

$$\Delta\omega = \left(\tfrac{1}{2}\right)\frac{C_1\omega_r^2 - C_2}{C_3\omega_r^2 + C_1\omega_r + C_4 + C_5\omega_r\sqrt{\omega_r^2+C_6}}, \tag{27}$$

where C_1, C_2, C_3, C_4, C_5 and C_6 are as defined previously. This is the final result of the derivation of bandwidth as a function of resonant frequency.

Experimental data was taken with a single crystal barium ferrite.[10] The ferrite was mounted as shown in Fig. 1 and data of bandwidth and applied frequency was taken between frequencies 58 and 59 kMc. The constants of (27) were adjusted to fit the experimental data. The results as seen in Fig. 2 show that bandwidth is increasing almost linearly with resonant frequency.

[10] $BaFe_{12}O_{19}$ supplied by the A. O. Smith Corp., Milwaukee, Wis.

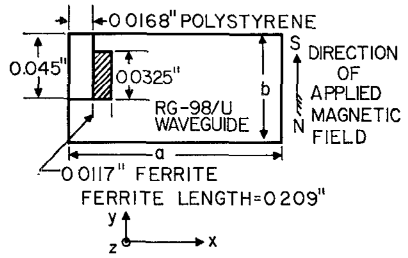

Fig. 1—Cross section of millimeter-wave resonance isolator.

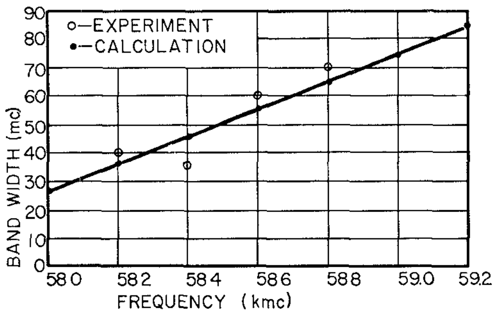

Fig. 2—Bandwidth of single crystal ferrite isolator vs resonant frequency. Calculated curve was obtained with

$$C_1 = 7.68 \times 10^{-24} \frac{\sec^2}{m^2}, \qquad C_2 = 1.0 \frac{1}{m^2},$$

$$C_3 = 2.97 \times 10^{-34} \frac{1}{\sec m^2}, \qquad C_4 = 1.48 \times 10^{-11} \frac{\sec}{m^2},$$

$$C_5 = 2.06 \times 10^{-37} \frac{\sec^3}{m^2}, \quad \text{and} \quad C_6 = -6.27 \times 10^{-22} \frac{1}{\sec^2}.$$

Line Width and Resonant Frequency

It is the purpose of this section to derive a relation between the line width and the resonant frequency of a single crystal ferrite isolator at millimeter wavelength. Kittel[4] developed a relation between applied field and resonant frequency, given by

$$\omega_r^2 = A_1 H_0^2 + A_2 H_0 + A_3, \tag{28}$$

where

$$A_1 = \gamma^2$$
$$A_2 = (N_x - N_y - 2N_z)\gamma^2 M + 2\gamma^2 H_a$$
$$A_3 = (N_x - N_z)(N_y - N_z)\gamma^2 M^2 + [(N_x - N_z) + (N_y - N_z)]\gamma^2 H_a M.$$

Line width, ΔH, is defined[1] as the difference between the two magnetic field values one on either side of resonance where the imaginary part of the diagonal component of the susceptibility tensor is half of its value at resonance.

Let $\Delta H = H_2 - H_1$, and H_0 equal the applied field at resonance. Then, assuming a symmetrical resonance curve,

$$H_2 = H_0 + \frac{\Delta H}{2}, \tag{29}$$

$$H_1 = H_0 - \frac{\Delta H}{2}, \tag{30}$$

$$H_2^2 = H_0^2\left(1 + \frac{\Delta H}{H_0} + \frac{\Delta H^2}{4H_0^2}\right). \tag{31}$$

The term $\Delta H^2/4H_0^2$ can be considered negligible in this case since for the material being considered, the line width is 1/100 of the applied field. Thus (31) becomes

$$H_2^2 = H_0^2 + \Delta H H_0, \tag{32}$$

and similarily

$$H_1^2 = H_0^2 - \Delta H H_0. \tag{33}$$

Putting (29) and (32) into (28)

$$\omega_2^2 = A_1(H_0^2 + \Delta H H_0) + A_2\left(H_0 + \frac{\Delta H}{2}\right) + A_3 \tag{34}$$

and putting (30) and (33) into (28)

$$\omega_1^2 = A_1(H_0^2 - \Delta H H_0) + A_2\left(H_0 - \frac{\Delta H}{2}\right) + A_3. \tag{35}$$

Subtracting (34) and (35)

$$\omega_2^2 - \omega_1^2 = \Delta H(2A_1 H_0 + A_2). \tag{36}$$

From (25)

$$2\omega_r \Delta\omega = \Delta H\left(\frac{1}{C_7}\right), \tag{37}$$

where

$$C_7 = \frac{1}{2A_1 H_0 + A_2}.$$

Using (27) the equation becomes

$$\Delta H = \frac{C_7(C_1\omega_r^3 - C_2\omega_r)}{C_3\omega_r^2 + C_1\omega_r + C_4 + C_5\omega_r\sqrt{\omega_r^2 + C_6}}. \tag{38}$$

This is the derived relation of line width and resonant frequency.

Using data obtained as before, the constant C_7 was adjusted to fit this data. The other constants C_1 to C_6 remain the same as in (27). The results are shown in Fig. 3. It is seen that with only one constant adjusted, (38) could predict the experimental data fairly closely. Line width is shown to increase almost linearly with resonant frequency.

A much simpler equation found in the literature[1] that describes the relation of line width and resonant frequency is given as

$$\Delta H = \frac{2\omega\alpha}{\gamma}, \tag{39}$$

where α is a dimensionless damping factor. It was found that this equation could not fit the experimental data.

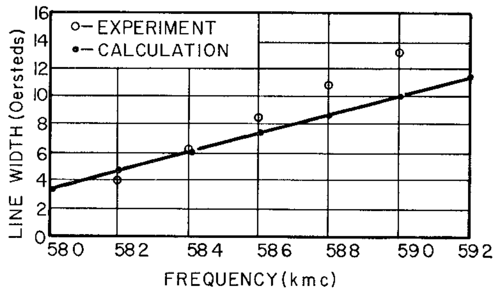

Fig. 3—Line width of single crystal ferrite isolator vs resonant frequency. Calculated curve was obtained with C_1, C_2, C_3, C_4, C_5, and C_6 the same as Fig. 2. $C_7=2.97\times10^{-20}$ oersted-sec².

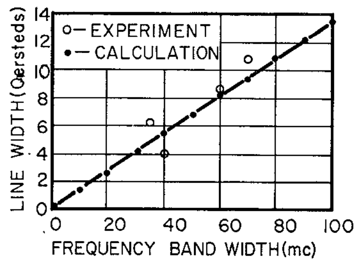

Fig. 4—Line width of a single crystal ferrite isolator vs frequency bandwidth.

Line Width and Bandwidth

The square root in the denominator of (27) can be simplified as follows:

$$\sqrt{\omega_r^2+C_6} \approx \omega_r\left(1+\frac{C_6}{2\omega_r^2}\right). \tag{40}$$

Replacing (40) into (27) and rearranging

$$\Delta\omega = \frac{C_1\omega_r^2 - C_2}{C_3'\omega_r^2 + 2C_1\omega_r + C_4'} \tag{41}$$

where

$$C_3' = 2(C_3 + C_5)$$
$$C_4' = 2C_4 + C_5C_6.$$

Rearranging the terms in (41) and solving for ω_r

$$W_r = \frac{-\Delta\omega C_1 \pm \sqrt{C_8\Delta\omega^2 + C_9\Delta\omega + C_1C_2}}{C_{10}\Delta\omega - C_1} \tag{42}$$

where

$$C_8 = C_1^2 - C_3'C_4'$$
$$C_9 = C_1C_4' - C_2C_3'$$
$$C_{10} = C_3'.$$

Substituting (42) into (37)

$$\Delta H = 2C_7\Delta\omega\left(\frac{-\Delta\omega C_1 \pm \sqrt{C_8\Delta\omega^2+C_9\Delta\omega+C_1C_2}}{C_{10}\Delta\omega - C_1}\right). \tag{43}$$

The constants C_8 to C_{10} are made up of constants that already have been determined. It was found that when the values for these constants were substituted into (43), many terms were negligible. So for the special case of the single crystal ferrite isolator at 58 kMc, (43) reduced to

$$\Delta H = \frac{2C_7}{\sqrt{C_1}}\Delta\omega. \tag{44}$$

This equation along with the experimental data obtained is shown in Fig. 4. It is seen that line width increases linearly for the range of frequencies where data was taken.

Line Width and Applied Field

A relation between line width and applied field can be obtained by substituting (41) into (37) and putting in Kittel's relation (28) wherever ω_r appears.

$$\Delta H = \frac{2C_7\sqrt{A_1H_0^2 + A_2H_0 + A_3}\,[C_1(A_1H_0^2 + A_2H_0 + A_3) - C_2]}{C_3'(A_1H_0^2 + A_2H_0 + A_3) + 2C_1\sqrt{A_1H_0^2 + A_2H_0 + A_3} + C_4'}. \tag{45}$$

The constants C_3', C_4', and C_7 in (45) are functions of the applied field. Therefore (45) becomes

$$\Delta H = \frac{[2/(2A_1H_0 + A_2)][C_1(A_1H_0^2 + A_2H_0 + A_3 - C_2]\sqrt{A_1H_0^2 + A_2H_0 + A_3}}{(Y_1H_0 + Y_2)(A_1H_0^2 + A_2H_0 + A_3) + 2C_1\sqrt{A_1H_0^2 + A_2H_0 + A_3} + Y_3H_0 + Y_4}, \tag{46}$$

where

$$C_3' = Y_1H_0 + Y_2$$
$$C_4' = Y_3H_0 + Y_4$$

and

$$Y_1 = \frac{T^2}{2}(S_1\epsilon_0\mu_0)$$

$$Y_2 = \frac{T^2}{2}\{S_3\sqrt{\epsilon_0\mu_0} - S_1\epsilon_0\mu_0[(N_y - N_z)\omega_m + \omega_a]$$

$$Y_3 = \frac{T^2\gamma}{2}(S_2 - S_1k^2)$$

$$Y_4 = \frac{T^2}{2}\left\{S_1k^2[\omega_m(N_y - N_z) + \omega_a] + S_2[\omega_m(N_x - N_z) + \omega_a] - \frac{S_3k^2}{\sqrt{\epsilon_0\mu_0}}\right\}.$$

Rearranging and multiplying out the denominator (46) becomes

$$\Delta H = \frac{\sqrt{A_1H_0^2 + A_2H_0 + A_3}\,[2C_1(A_1H_0^2 + A_2H_0 + A_3) - C_2]}{C_{11}H_0^4 + C_{12}H_0^3 + C_{13}H_0^2 + C_{14}H_0 + C_{15}H_0\sqrt{A_1H_0^2 + A_2H_0 + A_3} + C_{16}\sqrt{A_1H_0 + A_2H_0 + A_3} + C_{17}}, \quad (47)$$

where

$$C_{11} = 2Y_1A_1^2$$
$$C_{12} = 3Y_1A_1A_2 + Y_2A_1$$
$$C_{13} = A_2(Y_1A_2 + Y_2A_1) + 2A_1(Y_1A_3 + Y_2A_2 + Y_3)$$
$$C_{14} = 2A_1(Y_4 + Y_2A_3) + A_2(Y_1A_3 + Y_2A_2 + Y_3)$$
$$C_{15} = 4A_1C_1$$
$$C_{16} = 2A_2C_1$$
$$C_{17} = A_2(Y_4 + Y_2A_3).$$

The constants C_{11} to C_{17} were obtained by fitting (47) to experimental data. This equation, with adjusted constants and the experimental data, is shown in Fig. 5. The line width is seen to increase with the value of the applied field.

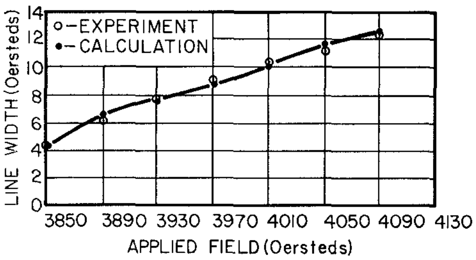

Fig. 5—Line width of a single crystal ferrite isolator vs applied magnetic field. Calculated curve was obtained with

$A_1 = 3.06 \times 10^{13} \frac{1}{(\text{oersted-sec})^2}$, $A_2 = 3.81 \times 10^{16} \frac{1}{\text{oersted-sec}^2}$,

$A_3 = 2.799 \times 10^{21} \frac{1}{\text{sec}^2}$, $C_{11} = 3.06 \times 10^{-2} \frac{1}{\text{m}^2\ \text{oersted}^5\text{-sec}}$,

$C_{12} = -4.28 \times 10^2 \frac{1}{\text{m}^2\ \text{oersted}^4\text{-sec}}$, $C_{13} = 2.08 \times 10^6 \frac{1}{\text{m}^2\ \text{oersted}^3\text{-sec}}$,

$C_{14} = -6.79 \times 10^9 \frac{1}{\text{m}^2\ \text{oersted}^2\text{-sec}}$, $C_{15} = 7.61 \times 10^{-3} \frac{1}{\text{m}^2\ \text{oersted}^2}$,

$C_{16} = -30.3 \frac{1}{\text{m}^2\ \text{oersted}}$, $C_{17} = 1.33 \times 10^{13} \frac{1}{\text{m}^2\ \text{oersted sec}}$.

Conclusions

A theoretical equation of bandwidth of a single crystal millimeter-wave resonance isolator was newly derived. In the region where experimental data was taken, the trend was an increase in bandwidth when the resonant frequency was increased. The theoretical equation agreed with the experimental results.

A theoretical equation of line width and resonant frequency of ferrite isolator was derived. The equation could predict the experiment well when only one constant out of the seven was adjusted to fit the experimental data. The equation and data indicated that line width increased almost linearly with resonant frequency in the range of frequencies considered.

A theoretical equation of line width and frequency bandwidth was derived. Although the general equation suggested a more complex relation, the equation reduced to a linear relation between line width and bandwidth when values for the constants were put into the equation. The experimental data agreed with the linear theoretical equation to some extent.

Finally, a theoretical equation of line width and applied field was derived. This equation proved to be more complex than the other derived equations. But again, the equation could be adjusted to follow quite closely the experimental results. The curve showed increasing line width with increasing applied field.

Acknowledgment

The authors wish to thank F. F. Y. Wang of the A. O. Smith Corporation for supplying the ferrite samples and for his invaluable discussion, the Raytheon Company for contributing the millimeter-wave tubes, and W. D. McCoy for building special switches for the isolator experiments. Thanks are also extended to S. Krupnik, Jr., J. A. Stefancin, and J. E. Billo for their assistance.